1) Find the first partial derivatives $f_x(x, y)$ and $f_y(x, y)$ of the function.

a) $f(x, y) = x^2 - 2y^2 + 4$

b) $f(x, y) = \dfrac{4x^3}{y^2}$

c) $f(x, y) = 2y^2\sqrt{x}$

d) $f(x, y) = y^3 - 2xy^2 - 1$

e) $z = e^{x/y}$

f) $z = ye^{y/x}$

g) $z = \ln\sqrt{xy}$

h) $z = \ln\dfrac{x+y}{x-y}$

i) $z = \dfrac{xy}{x^2 + y^2}$

j) $h(x, y) = e^{-(x^2+y^2)}$

k) $z = \cos xy$

l) $z = \sin 5x \cos 5y$

m) $z = e^y \sin xy$

n) $f(x, y) = \int_x^y (t^2 - 1)\, dt$

a) $f_x(x, y) = 2xy^3,\ f_y(x, y) = 3x^2y^2$

b) $f_x(x, y) = 12x^2y^{-2},\ f_y(x, y) = -8x^3y^{-3}$

c) $\dfrac{\partial z}{\partial x} = \dfrac{y^2}{\sqrt{x}},\quad \dfrac{\partial z}{\partial y} = 4y\sqrt{x}$

d) $\dfrac{\partial z}{\partial x} = -2y^2,\quad \dfrac{\partial z}{\partial y} = 3y^2 - 4xy$

e) $\dfrac{\partial z}{\partial x} = \dfrac{1}{y}e^{x/y},\quad \dfrac{\partial z}{\partial y} = \dfrac{-x}{y^2}e^{x/y}$

f) $\dfrac{\partial z}{\partial x} = -\dfrac{y^2}{x^2}e^{y/x},\quad \dfrac{\partial z}{\partial y} = e^{y/x}\left(1 + \dfrac{y}{x}\right)$

g) $\dfrac{\partial z}{\partial x} = \dfrac{1}{2x},\quad \dfrac{\partial z}{\partial y} = -\dfrac{1}{y}$

h) $\dfrac{\partial z}{\partial x} = \dfrac{-2y}{(x+y)(x-y)},\quad \dfrac{\partial z}{\partial y} = \dfrac{2x}{(x+y)(x-y)}$

i) $\dfrac{\partial z}{\partial x} = \dfrac{y^3 - x^2y}{(x^2+y^2)^2},\quad \dfrac{\partial z}{\partial y} = \dfrac{x^3 - xy^2}{(x^2+y^2)^2}$

j) $h_x(x, y) = -2xe^{-(x^2+y^2)},\quad h_y(x, y) = -2ye^{-(x^2+y^2)}$

k) $\dfrac{\partial z}{\partial x} = -y\sin xy,\quad \dfrac{\partial z}{\partial y} = -x\sin xy$

l) $\dfrac{\partial z}{\partial x} = 5\cos 5x \cos 5y,\quad \dfrac{\partial z}{\partial y} = -5\sin 5x \sin 5y$

m) $\dfrac{\partial z}{\partial x} = ye^y\cos xy,\quad \dfrac{\partial z}{\partial y} = e^y(x\cos xy + \sin xy)$

n) $f_x(x, y) = 1 - x^2,\ f_y(x, y) = y^2 - 1$

2) Use the limit definition of partial derivatives to find $f_x(x, y)$ and $f_y(x, y)$.

a) $f(x, y) = 3x + 2y$

b) $f(x, y) = \dfrac{1}{x+y}$

a) $f_x(x, y) = 3,\ f_y(x, y) = 2$

b) $f_x(x, y) = \dfrac{-1}{(x+y)^2},\ f_y(x, y) = \dfrac{-1}{(x+y)^2}$

3) Evaluate f_x and f_y at the given point.

a) $f(x,y)=\sin xy,\ \left(2,\dfrac{\pi}{4}\right)$

b) $f(x,y)=\dfrac{2xy}{\sqrt{4x^2+5y^2}},\ (1,1)$

a) $\boxed{f_x\left(2,\dfrac{\pi}{4}\right)=0,\ f_y\left(2,\dfrac{\pi}{4}\right)=0}$

b) $\boxed{f_x(1,1)=\dfrac{10}{27},\ f_y(1,1)=\dfrac{8}{27}}$

4) Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

a) $x^2+y^2+z^2=3xyz$

b) $x-z=\arctan(yz)$

a) $\boxed{\dfrac{\partial z}{\partial x}=\dfrac{3yz-2x}{2z-3xy},\quad \dfrac{\partial z}{\partial y}=\dfrac{3xz-2y}{2z-3xy}}$

b) $\boxed{\dfrac{\partial z}{\partial x}=\dfrac{1+y^2z^2}{1+y+y^2z^2},\quad \dfrac{\partial z}{\partial y}=-\dfrac{z}{1+y+y^2z^2}}$

5) Find the first partial derivatives with respect to x, y and z.

a) $f(x,y,z)=3x^2y-5xyz+10yz^2$

b) $w=\dfrac{7xz}{x+y}$

a) $\boxed{f_x(x,y,z)=6xy-5yz,\ f_y(x,y,z)=3x^2-5xz+10z^2,\ f_z(x,y,z)=-5xy+20yz}$

b) $\boxed{\dfrac{\partial w}{\partial x}=\dfrac{7yz}{(x+y)^2},\quad \dfrac{\partial w}{\partial y}=\dfrac{-7xz}{(x+y)^2},\quad \dfrac{\partial w}{\partial z}=\dfrac{7x}{x+y}}$

6) Evaluate f_x, f_y, and f_z at the given point.

a) $f(x,y,z)=\dfrac{xy}{x+y+z},\ (3,1,-1)$

b) $f(x,y,z)=z\sin(y+x),\ \left(0,\dfrac{\pi}{2},-4\right)$

a) $\boxed{f_x(3,1,-1)=0,\ f_y(3,1,-1)=\dfrac{2}{3},\ f_z(3,1,-1)=-\dfrac{1}{3}}$

b) $\boxed{f_x\left(0,\dfrac{\pi}{4},-4\right)=0,\ f_y\left(0,\dfrac{\pi}{4},-4\right)=0,\ f_z\left(0,\dfrac{\pi}{4},-4\right)=1}$

7) Show that the mixed partials $\dfrac{\partial^2 z}{\partial x \partial y}$ and $\dfrac{\partial^2 z}{\partial y \partial x}$ are equal. (Verify that the conclusion of Clairaut's theorem holds.)

a) $z = x^4 - 3x^2 y^2 + y^4$

b) $z = 2xe^y - 3ye^{-x}$

a) $\boxed{\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x} = -12xy}$

b) $\boxed{\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x} = 2e^y + 3e^{-x}}$

8) For $f(x, y)$, find all values of x and y such that $f_x(x, y) = 0$ and $f_y(x, y) = 0$ simultaneously.

a) $f(x, y) = x^2 + xy + y^2 - 2x + 2y$

b) $f(x, y) = e^{x^2 + xy + y^2}$

a) $\boxed{(2, -2)}$

b) $\boxed{(0, 0)}$

9) Find the indicated partial derivative.

a) $f(r, s, t) = r \ln(rs^2 t^3)$; f_{rss}, f_{rst}

b) $z = u\sqrt{v - w}$; $\dfrac{\partial^3 z}{\partial u \partial v \partial w}$

a) $\boxed{f_{rss} = -2s^{-2}, \ f_{rst} = 0}$

b) $\boxed{\dfrac{\partial^3 z}{\partial u \partial v \partial w} = \dfrac{1}{4}(v - w)^{-3/2}}$

10) Show that the function $z = \cos(4x + 4ct)$ satisfies the wave equation $\dfrac{\partial^2 z}{\partial t^2} = c^2 \dfrac{\partial^2 z}{\partial x^2}$.

Show